This is a report on joint work with David Smith and Hipolito Treffinger.

The notion of (semi-)stability has been introduced in representation theory of quivers by Schofield and King, and it was formalised in the context of abelian categories by Rudakov. The concept has re-appeared in mathematical physics as scattering diagrams, and the same wall-and-chamber structure is also studied in the work of Bridgeland. It seems very natural to join two recent developments, the wall-and-chamber structure of scattering diagrams with the combinatorial structure of the fan associated with $\tau$-tilting modules as described by Demonet, Iyama and Jasso. In fact, we learned that David Speyer and Hugh Thomas were independently following similar ideas.

We explain in the talk how the $\tau$—tilting fan can be embedded into King’s stability manifold: Each support $\tau$-tilting pair $(M, P)$ yields a chamber $C_{(M,P)}$, and one can give a complete description of the walls bordering this chamber $C_{(M,P)}$. Moreover, we associate to each chamber $C$ a torsion class $T_C$.

We further introduce and study the notion of green paths, which can be seen as a continuous version of the maximal green sequences introduced by Keller in cluster theory in order to study Donaldson-Thomas invariants.