A Tangled Approach to Cross Product Algebras, Their Invariants and Centralizers

An algebra $V$ with a cross product has dimension 3 or 7. In this talk, we describe how 3-tangles can provide a basis for the space of homomorphisms from $V^\otimes n$ to $V^\otimes m$ which are invariant under the action of the automorphism group $G$ of $V$. The group $G$ is a special orthogonal group when $\dim V = 3$ and a simple algebraic group of type $G_2$ when $\dim V = 7$. When $m = n$, this gives a graphical description of the centralizer algebra $\text{End}_G(V^\otimes n)$, and hence also a graphical realization of the $G$-invariants in $V^\otimes 2n$ equivalent to the First Fundamental Theorem of Invariant Theory. Our approach using certain properties of the cross product differs from that of Kuperberg, which derives quantum $G_2$-link invariants from the Jones polynomial starting from its simplest formulation in terms of the Kauffman bracket. The 3-dimensional simple Kaplansky Jordan superalgebra can be interpreted as a cross product (super)algebra, and 3-tangles can be used to obtain a graphical description of its invariants and centralizer algebras relative to the action of the special orthosymplectic group. This is joint work with A. Elduque.