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A Tangled Approach to Cross Product Algebras, Their Invariants and Centralizers

An algebra V with a cross product has dimension 3 or 7. In this talk, we describe how 3-tangles can provide a basis for the space of homomorphisms from $\mathsf{V}^{\otimes n}$ to $\mathsf{V}^{\otimes m}$ which are invariant under the action of the automorphism group G of V. The group G is a special orthogonal group when $\dim \mathsf{V} = 3$ and a simple algebraic group of type G_2 when $\dim \mathsf{V} = 7$. When $m = n$, this gives a graphical description of the centralizer algebra $\mathsf{End}_{\mathsf{G}}(\mathsf{V}^{\otimes n})$, and hence also a graphical realization of the G-invariants in $\mathsf{V}^{\otimes 2n}$ equivalent to the First Fundamental Theorem of Invariant Theory. Our approach using certain properties of the cross product differs from that of Kuperberg, which derives quantum G_2-link invariants from the Jones polynomial starting from its simplest formulation in terms of the Kauffman bracket. The 3-dimensional simple Kaplansky Jordan superalgebra can be interpreted as a cross product (super)algebra, and 3-tangles can be used to obtain a graphical description of its invariants and centralizer algebras relative to the action of the special orthosymplectic group. This is joint work with A. Elduque.