Some invariants of the super Jordan plane

Hochschild cohomology and its Gerstenhaber algebra structure are relevant Morita, tilting and derived invariants. Their computation requires a resolution of the algebra as a bimodule over itself. There is always a canonical resolution available, the bar resolution, useful from a theoretical point of view, but not in practice: its complexity rarely allows explicit calculations.

Nichols algebras are generalizations of symmetric algebras in the context of braided tensor categories. They are fundamental objects for the classification of pointed Hopf algebras. Heckenberger classified finite-dimensional Nichols algebras of diagonal type up to isomorphism. The classification separates the Nichols algebras into three different classes. Later, Angiono described the defining relations of the Nichols algebras of Heckenberger’s list.

In a joint work with Sebastián Reca, we computed the Hochschild (co)homology of $A = k\langle x, y \rangle / (x^2, y^2 x - xy^2 - xyx)$ – the super Jordan plane –, when $\text{char}(k) = 0$. This algebra is the Nichols algebra $B(V(-1, 2))$, of Gelfand-Kirillov dimension 2.

Our main results are the following:

1) We give explicit bases for the Hochschild (co)homology spaces.

2) We describe the cup product and we thus see that the isomorphism between $H^{2p}(A, A)$ and $H^{2p+2}(A, A)$, where $p > 0$, is given by the multiplication by an element of $H^2(A, A)$, and similarly for the odd degrees.

3) We describe the Lie algebra structure of $H^1(A, A)$, which is isomorphic to a Lie subalgebra of the Virasoro algebra.