A monitoring approach to recurrence in quantum walks, based on checking the return after every step, has been proposed recently. A central role is played by the generating function of first returns, which turns out to be a so called Schur function, a well known mathematical object which links this quantum topic to areas like approximation theory, harmonic analysis, operator theory, orthogonal polynomials or system theory. This has been the origin of a rich interplay which allowed for a spectral characterization of quantum recurrence, but also provided new insights into unsolved mathematical problems.

A more recent discovery connects the first return generating functions to the study of symmetry protected topological phases in quantum walks. Such generating functions codify the relevant information of the evolution operator to identify the corresponding topological phases. This reduces the computation of symmetry protected topological indices to simple finite-dimensional Linear Algebra problems, allowing for a complete classification of topological phases for arbitrary non-translation invariant coined walks and generalizations.

This talk will be a review of the above results, including the geometric and topological meaning that the first return generating functions give to certain recurrence properties, or the striking behaviour of such generating functions under certain factorizations of quantum walks, which lead to recurrence splitting rules for quantum walks.

The results presented in this talk are the result of joint collaborations with R.F. Werner, C. Cedzich, C. Stahl, T. Geib (U Hannover), A.H. Werner (U Copenhagen), F.A. Grünbaum, J. Wilkening (UC Berkeley) and J. Bourgain (IAS Princeton).