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The strange geometry of high-dimensional random forests

The uniform spanning forest (USF) in the lattice $\mathbb{Z}^d$, first studied by Pemantle, is defined as a limit of uniform spanning trees in growing finite boxes. Although the USF is a limit of trees, it might not be connected- Indeed, Pemantle proved that the USF in $\mathbb{Z}^d$ is connected if and only if $d < 5$. Later, Benjamini, Kesten, Peres and Schramm extended this result, and showed that the component structure of the USF undergoes a phase transition every 4 dimensions: For dimensions d between 5 and 8 there are infinitely many trees, but any two trees are adjacent; for d between 9 and 12 this fails, but for every two trees in the USF there is an intermediary tree, adjacent to each of the them. This pattern continues, with the number of intermediary trees required increasing by 1 every 4 dimensions. In this talk, I will show that this is not the whole story, and for $d > 8$ the USF geometry undergoes a qualitative change every time the dimension increases by 1.

Joint work with Yuval Peres.