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Minimal Generating Sets of the Symmetric Group

The goal of this project is to analyze the minimal generating sets of the symmetric group S_n . To accomplish the task, we first build all of them for S_3 through S_5 . We use group automorphisms and cycle-type to facilitate this. Specifically, we organize our search for minimal generating sets by the cycle-types of its elements, and we identify any such X with any of its images under conjugation. As such, "orbit size" becomes the first interesting aspect of the project. Given a minimal generating set X from an orbit, we construct the rooted tree such that each node is an element w of S_n . Its path to the root represents a shortest expression for w in terms of the generators. The properties (such as depth and width) of such trees, uniqueness up to automorphism, the posets of minimal generating sets not of the form $\{(1, 2), (2, 3), \dots, (n - 1, n)\}$, and the minimal generating sets (for $n = 3, 4, 5$) that fit into a family for any $n \geq 3$ are studied. (Preliminary report of work started under the auspices of the McNair program at Loyola Chicago.)