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Toeplitz Quantization

I discuss some recent work with Wolfram Bauer and Raffael Hagger. Here, C^n is complex n -space and, for z in C^n , we consider the standard family of Gaussian measures $d\mu_t(z) = (4\pi t)^{-n} \exp(-|z|^2/4t) dv(z)$, $t > 0$ where dv is Lebesgue measure. We consider the Hilbert space L_t^2 of all μ_t -square integrable complex-valued measurable functions on C^n and the closed subspace of all square-integrable entire functions, H_t^2 . For f measurable and h in H_t^2 with fh in L_t^2 , we consider the Toeplitz operators $T_f^{(t)}h = P^{(t)}(fh)$ where $P^{(t)}$ is the orthogonal projection from L_t^2 onto H_t^2 . For bounded f (f in L^∞) and some unbounded f , these are bounded operators with norm $\|\cdot\|_t$. For f, g bounded, with "sufficiently many" bounded derivatives, there are known deformation quantization conditions, including (0) $\lim_{t \rightarrow 0} \|T_f^{(t)}\|_t = \|f\|_\infty$ and (1) $\lim_{t \rightarrow 0} \|T_f^{(t)}T_g^{(t)} - T_{fg}^{(t)}\|_t = 0$. We exhibit a pair of bounded real-analytic functions F, G so that (1) fails. On the positive side, for the space VMO of functions with vanishing mean oscillation, we show that (1) holds for all f in (the sup-norm closed algebra) $VMO \cap L^\infty$ and g in L^∞ . (1) also holds for all f in UC (uniformly continuous functions, bounded or not) while (0) holds for all bounded continuous f .