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Commutative algebras of Toeplitz operators on the Siegel domain

We describe several ways of how the symbols, subordinated to the nilpotent group of biholomorphisms of the unit ball (i.e., invariant under the action of a subgroup of the nilpotent group), generate Banach (and even C^*) algebras that are commutative on each weighted Bergman space.

Recall for completeness that the nilpotent group of biholomorphisms of the Siegel domain D_n , the unbounded realization of the unit ball in \mathbb{C}^n , is isomorphic to $\mathbb{R}^{n-1} \times \mathbb{R}_+$ with the following group action

$$(b, h) : (z', z_n) \in D_n \mapsto (z' + b, z_n + h + 2iz' \cdot b + i|b|^2) \in D_n,$$

for each $(b, h) \in \mathbb{R}^{n-1} \times \mathbb{R}_+$.

The key role in our study is played by the direct integral decomposition of the isomorphic image of the Bergman space on the Siegel domain, which is direct integral where each component is a weighted Fock spaces. We describe the action of Toeplitz operators with certain symbols as a direct integral of scalar multiplication operators and a direct integral of Toeplitz operators with the same symbol on the weighted Fock spaces. Note that all the above symbols are invariant under the action of the subgroup $\mathbb{R}^\ell \times \mathbb{R}_+$ ($\ell < n - 1$) of the nilpotent group.