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Sharp convergence rates for averaged non expansive maps

We consider the well-known Kranoselskii-Mann (KM) sequential averaging iteration

$$x_{n+1} = (1-\alpha_{n+1})\, x_n + \alpha_{n+1}\, T x_n, \tag{KM}$$

where $T : C \to C$ is nonexpansive with C bounded closed convex in a Banach space, and $0 \le \alpha_n \le 1$. This provides a method to compute fixed points and to establish their existence, and it finds many applications in Optimization and beyond.
A key step in proving the convergence of the iteration is to show that $\|x_n - Tx_n\| \to 0$ as $n \to \infty$. This property is named *asymptotic regularity* and has been studied under different assumptions on the space and the set C. In 1976, Browder and Petryshyn showed that asymptotic regularity holds in every Banach space, provided $\sum_i \alpha_i = +\infty$. In 1992, Baillon and Bruck conjectured a bound that implies all the previous results: there exists a universal constant κ such that

$$\|x_n - Tx_n\| \le \kappa \frac{\mathsf{diam}(C)}{\sqrt{\sum_{i=1}^{n} \alpha_i (1-\alpha_i)}}. \tag{B}$$

Cominetti et al. (2014) proved that (B) holds with $\kappa = 1/\sqrt{\pi}$. In this talk we show that this constant is sharp. Our approach builds on the idea of constructing a sequence of bounds $\|x_m - x_n\| \le d_{mn}$ for $m \le n$, which are valid for every nonexpansive map. The d_{mn}'s are defined by a recursive sequence of discrete optimal transport problems, and can be interpreted as absorption probabilities of a suitable Markov process.

We also discuss how to extend this approach to establish new rates of convergence for inexact versions of the (KM) iteration.

Based on joint work with Roberto Cominetti and Matias Pavez-Signe.