We consider the issue of energy conservation for weak solutions of the 2D Euler system with an $L^p$-control on vorticity, for some $p \geq 1$. This is related to the Onsager conjecture, recently established, which states that, for 3D flows, energy is conserved if and only if the velocity is $1/3$-Holder continuous. The Onsager critical regularity is valid in any dimension, however, the forward enstrophy cascade expected in turbulent solutions, from Kraichnan’s 2D turbulence theory, suggests there may be a regularizing effect not seen in 3D. It is, hence, plausible that there be a (dynamical) mechanism preventing anomalous energy dissipation in 2D, even for solutions that are not a priori $1/3$ regular, which cannot be seen by simply estimating the energy flux.

We use a direct argument, based on a mollification in physical space, to show that energy of a weak solution is conserved if $\omega = \nabla \perp \cdot u \in L^{3/2}$. We construct an example of a 2D field $u \in B^{\frac{1}{3}/3, \infty}$ (an Onsager-critical space), whose 2D-curl belongs to $L^{3/2-\varepsilon}$, for any $\varepsilon > 0$, such that the energy flux is non-vanishing, thereby establishing sharpness of the kinematic argument. Finally, we prove that any solution to the Euler equations produced via a vanishing viscosity limit from the Navier-Stokes equations, with $\omega \in L^p$, for $p > 1$, conserves energy. We call such solutions physically realizable, and we conclude that there is, indeed, a mechanism preventing anomalous dissipation in 2D in Onsager supercritical spaces.