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Critical Regularity for Energy Conservation in 2D Inviscid Fluid Dynamics

We consider the issue of energy conservation for weak solutions of the $2D$ Euler system with an L^p-control on vorticity, for some $p \geq 1$. This is related to the Onsager conjecture, recently established, which states that, for $3D$ flows, energy is conserved if and only if the velocity is $1/3$-Holder continuous. The Onsager critical regularity is valid in any dimension, however, the forward enstrophy cascade expected in turbulent solutions, from Kraichnan's $2D$ turbulence theory, suggests there may be a regularizing effect not seen in $3D$. It is, hence, plausible that there be a (dynamical) mechanism preventing anomalous energy dissipation in $2D$, even for solutions that are not a priori $1/3$ regular, which cannot be seen by simply estimating the energy flux.

We use a direct argument, based on a mollification in physical space, to show that energy of a weak solution is conserved if $\omega = \nabla^\perp \cdot u \in L^{3/2}$. We construct an example of a $2D$ field $u \in B^{1/3}_{3,\infty}$ (an Onsager-critical space), whose $2D$-curl belongs to $L^{3/2-\varepsilon}$, for any $\varepsilon > 0$, such that the energy flux is non-vanishing, thereby establishing sharpness of the kinematic argument. Finally, we prove that any solution to the Euler equations produced via a vanishing viscosity limit from the Navier-Stokes equations, with $\omega \in L^p$, for $p > 1$, conserves energy. We call such solutions *physically realizable*, and we conclude that there is, indeed, a mechanism preventing anomalous dissipation in 2D in Onsager supercritical spaces.