Towers of nodal bubbles for the Bahri-Coron problem in punctured domains

Let $\Omega$ be a bounded smooth domain in $\mathbb{R}^N$ which contains a ball of radius $R$ centered at the origin, $N \geq 3$. Under suitable symmetry assumptions, for each $\delta \in (0, R)$, we establish the existence of a sequence $(u_{m, \delta})$ of nodal solutions to the critical problem

$$
\begin{cases}
-\Delta u = |u|^{2^* - 2}u & \text{in } \Omega_\delta := \{x \in \Omega : |x| > \delta\}, \\
u = 0 & \text{on } \partial \Omega_\delta,
\end{cases}
$$

where $2^* := \frac{2N}{N-2}$ is the critical Sobolev exponent. We show that, if $\Omega$ is strictly starshaped, then, for each $m \in \mathbb{N}$, the solutions $u_{m, \delta}$ concentrate and blow up at 0, as $\delta \to 0$, and their limit profile is a tower of nodal bubbles, i.e., it is a sum of rescaled nonradial sign-changing solutions to the limit problem

$$
\begin{cases}
-\Delta u = |u|^{2^* - 2}u, \\
u \in D^{1,2}(\mathbb{R}^N),
\end{cases}
$$
centered at the origin.

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