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*Kurdyka-Łojasiewicz-Simon inequality for gradient flows in metric spaces*

The classical Łojasiewicz inequality and its extensions by Simon and Kurdyka have been a considerable impact on the analysis of the large time behaviour of gradient flow in Hilbert spaces. Our aim is to adapt the classical Kurdyka-Łojasiewicz and Łojasiewicz-Simon inequalities to the general framework gradient flow in metric spaces. We show that the validity of a Kurdyka-Łojasiewicz inequality imply trend to equilibrium in the metric sense, and the Kurdyka-Łojasiewicz inequality has the advantage to derive decay estimates of the trend to equilibrium and finite time of extinction. Also we study the relation between Kurdyka-Łojasiewicz inequality and the existence of talweg. The entropy method have proved to be very useful to study the large time behaviour of solutions to many EDP's. This method is based in the entropy-entropy production/disipation (EEP) inequality, which correspond to Kurdyka-Łojasiewicz inequality, and also in the entropy transportation (ET) inequality. We show that for geodesically convex functionals Kurdyka-Łojasiewicz inequality and entropy transportation (ET) inequality are equivalent. We apply our general results to gradient flow in Banach spaces and in spaces of probability measures with Wasserstein distances. For the energy functional associated with a doubly-nonlinear equations on $\mathcal{R}^N$ we obtain the equivalence between Łojasiewicz-Simon inequality, generalized log-Sobolev inequality and $p$-Talagrand inequality; also we get decay estimates for its solutions. Finally we apply our results to metric spaces with Ricci curvature bounds from below, getting that, in this context, a $p$-Talagrand inequality is equivalent to a Łojasiewicz-Simon inequality.

Joint work with Daniel Hauer (Sydney University)