Let $\mathcal{G}$ be an algebraic quantum group in the sense of Van Daele. We define a bivariant $K$-theory on the category of $\mathcal{G}$-module algebras. For each pair $(A, B)$ of $\mathcal{G}$-module algebras we define a group $\text{kk}^\mathcal{G}(A, B)$ and consider the category $\mathcal{KK}^\mathcal{G}$ whose objects are the $\mathcal{G}$-module algebras and the morphisms from $A$ to $B$ are the elements of $\text{kk}^\mathcal{G}(A, B)$.

Consider the functor $j^\mathcal{G} : \mathcal{G}-\text{Alg} \to \mathcal{KK}^\mathcal{G}$ which at the level of objects is the identity and at the level of morphisms sends $f : A \to B$ to its class $[f]$ in $\text{kk}^\mathcal{G}(A, B)$. The category $\mathcal{KK}^\mathcal{G}$ is triangulated and $j^\mathcal{G}$ is an excisive, homotopy invariant and $\mathcal{G}$-stable functor. Moreover, it is the universal functor for these properties.

The Green-Julg Theorem in Kasparov $KK$-theory states that if $G$ is a compact group and $B$ is a $G$-$C^*$-algebra, then there exists an isomorphism

$$\mu : KK^G(C, B) \to KK(C, B \rtimes G).$$

The main theorem of this talk is a version of Green-Julg Theorem for $\mathcal{KK}^\mathcal{G}$ when $\mathcal{G}$ is a semisimple Hopf algebra. Let $A$ be an algebra and $B$ a $\mathcal{G}$-module algebra then there exists an isomorphism

$$\psi : \text{kk}^\mathcal{G}(A^\tau, B) \to \text{kk}(A, B\#\mathcal{G})$$

where $B\#\mathcal{G}$ denotes the smash product. In particular, we obtain that $\text{kk}^\mathcal{G}(C, B) \simeq \text{KH}(B\#\mathcal{G})$. 