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State Sum Invariants of Three Manifolds from Spherical Multi-fusion Categories

We define a family of quantum invariants of closed oriented 3-manifolds using spherical multi-fusion categories. The state sum nature of this invariant leads directly to $(2+1)$ -dimensional topological quantum field theories (TQFTs), which generalize the Turaev-Viro-Barrett-Westbury (TVBW) TQFTs from spherical fusion categories. The invariant is given as a state sum over labeled triangulations, which is mostly parallel to, but richer than the TVBW approach in that here the labels live not only on 1-simplices but also on 0-simplices. It is shown that a multi-fusion category in general cannot be a spherical fusion category in the usual sense. Thus we introduce the concept of a spherical multi-fusion category by imposing a weakened version of sphericity. Besides containing the TVBW theory, our construction also includes the recent higher gauge theory $(2+1)$ -TQFTs given by Kapustin and Thorngren, which was not known to have a categorical origin before.