The occurrence of zero modes for Dirac operators with magnetic fields is the cause of break down of stability of matter for charged systems. All known examples of magnetic fields leading to zero modes are geometrically very complex. In order to better understand this geometry I will discuss singular magnetic fields supported on a finite number of possibly interlinking field lines (magnetic links). I will show that the occurrence of zero modes is intimately connected to the twisting and interlinking of the field lines. The result will rely on explicitly calculating appropriate spectral flows for the Dirac operators. This is joint work with Fabian Portmann and Jeremy Sok.