PHILLIP GRIFFITHS, Institute for Advanced Study
Hodge Theory and Moduli

The equivalence classes of smooth algebraic varieties X of a particular type form its moduli space $\mathcal{M}$, and their study is a central problem in algebraic geometry. When X is of general type $\mathcal{M}$ exists and has a canonical compactification $\overline{\mathcal{M}}$ as a projective algebraic variety. Aside from a few classical cases (curves, K3 surfaces, abelian varieties) very little is known about the boundary $\partial\mathcal{M} = \overline{\mathcal{M}}\backslash\mathcal{M}$ and the singular varieties X_0 that corresponds to boundary points. In this talk we will explain how Hodge theory provides basic invariants of the X_0's and in some early examples may be used to help understand geometrically the boundary structure of moduli.

Specifically, we will discuss how to define the Satake-Baily-Borel compactification $\overline{\mathcal{M}}_{SBB} = \ Proj\ \Omega_e$ of the image of the period mapping of $\mathcal{M}$ where $\Omega \to \overline{\mathcal{M}}$ is the canonically extended Hodge bundle. We will also discuss the examples where X is a regular surface with $p_g(X) = 2$ and $K_X^2 = 1, 2$; in these examples the boundary strata of $\partial M = \overline{\mathcal{M}}_{SBB}\backslash\mathcal{M}$ classify the Gorenstein degenerations of the X_0's. The above is joint work with Green, Laza and Robles, and the $K_X^2 = 1$ case uses recent results of Franciosi-Pardini and Rollenske.