Diophantine Equations with Binomials Coefficients and Perturbations of Symmetric Boolean Functions

This work presents a study of perturbations of symmetric Boolean functions. In particular, it establishes a connection between exponential sums of these perturbations and Diophantine equations of the form

\[ \sum_{l=0}^{n} \binom{n}{l} x_l, \]

where \( x_l \) belongs to some fixed bounded subset \( \Gamma \) of \( \mathbb{Z} \). The concepts of trivially balanced symmetric Boolean function and sporadic balanced Boolean function are extended to this type of perturbations. An observation made by Canteaut and Videau for symmetric Boolean functions of fixed degree is extended. To be specific, it is proved that, excluding the trivial cases, balanced perturbations of fixed degree do not exist when the number of variables grows. Some sporadic balanced perturbations are presented. Finally, a beautiful but unexpected identity between perturbations of two very different symmetric Boolean functions is also included in this work. This is a joint work with Oscar Gonzalez and Luis Medina.