Random graph processes

For the past several decades, since Erdős’ 1947 lower bound on the Ramsey numbers $R(k)$, randomness has been an important and powerful tool for demonstrating the existence of counter-intuitive objects. When dealing with random objects, it is often useful to reveal the randomness gradually, rather than all at once; that is, to turn a static random object into a random process. In the 1980s and 1990s, several important techniques for studying the evolution of such processes were introduced by Bollobás, Rödl, Ruciński, Wormald, and others, and in recent years these techniques have been developed further by a number of different authors, and have been used to resolve several well-known open problems.

In this talk we will describe a few of these recent developments, focusing our attention on two or three specific examples. In particular, we will discuss the Ramsey numbers $R(3, k)$, and a problem of Pomerance about the existence in a random set of integers of a subset whose product is a square. In each case, the key to the proof is controlling the evolution of a suitably-chosen random (hyper)graph process using self-correcting martingales.

This talk is based on joint work with Paul Balister, Béla Bollobás, Gonzalo Fiz Pontiveros, Simon Griffiths and Paul Smith.