Using certain non-uniqueness results for the Yamabe problem as motivation, I will describe deformations of compact flat manifolds and orbifolds. Flat orbifolds (resp., manifolds) are quotients of Euclidean spaces by crystallographic (resp., torsion-free crystallographic) groups. I will explain the basic structure of these objects and describe the space of deformations of flat metrics on them (Teichmüller space), showing that flat manifolds can always be deformed, while flat orbifolds may be rigid. I will also describe the boundary of the moduli space; showing that limits of flat manifolds are flat orbifolds and, conversely, that every flat orbifold is the limit of flat manifolds. This is joint work with Renato Bettiol and Andrzej Derdzinski.