Normal functions are “well-behaved” sections of a bundle of complex tori (intermediate Jacobians) associated to a period map (variation of Hodge structure). They arise from families of (homologically trivial) algebraic cycles on the fibers of a smooth proper morphism of varieties, and were first studied by Poincaré and Lefschetz for families of divisors on curves. Conversely, given a normal function, existence of such families of cycles is predicted by the Hodge Conjecture, one of the seven Millenium Problems.

A more general notion of cycles, due to Bloch and Beilinson and closely related to algebraic K-theory and motivic cohomology, leads to generalizations called “higher normal functions”. Normal functions and their higher analogues often show up in unexpected places, explaining and generalizing observed arithmetic and functional properties of periods.

In this talk, we will give a brief tour of these ostensibly “well-behaved” functions’ party-crashing exploits, from irrationality proofs and Apéry constants to quantum curves and Feynman amplitudes. No knowledge of algebraic cycles will be assumed.