We show how to carry out a sieve of Erastosthenes up to $N$ in space $O(N^{1/3})$ and essentially linear time. This improves over the usual versions, which take space about $O(\sqrt{N})$ and essentially linear time. The algorithm – which, like the one in (Galway, 2000), is ultimately related to diophantine approximation – can also be used to factorize integers $n$, and thus to give the values of arithmetical functions such as the Möbius function $\mu$ and the Liouville function $\lambda$ for all integers up to $N$. 