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A survey of first-order logic with dependent sorts (FOLDS)

FOLDS was introduced in the unpublished monograph I wrote in 1995; see the third item under "Papers" on my website. The last four sections of the fourth item, a paper published in 1998, gives a short introduction without proofs. The syntax of FOLDS is a simplified and generalized version of that of Per Martin-Lof's type theory. It is parametrized by a general concept of "FOLDS signature", a slightly more involved version of the Tarskian signature (similarity type) used in model theory. The main novelty is the concept of FOLDS-equivalence, a generalization of the concept of isomorphism. It seems that all concepts of "equivalence" in category theory, including higher categories, are special cases of FOLDS-equivalence. A rough statement of the main meta-theorem, the invariance theorem, proved in the monograph using tools of model theory both for classical and (in a suitable version) for intuitionistic logic, says that the first-order properties of structures of a fixed FOLDS-signature invariant under FOLDS-equivalence are exactly the ones that can be formulated in the FOLDS syntax. There are infinitary generalizations of the invariance theorem. The main applications of FOLDS are to higher-dimensional categories; see the item "The omega-category of all multitopic omega-categories; corrected" on the website. The general model-theory of FOLDS includes an elegant generalization of Per Lindstrom's classical theorem; it characterizes FOLDS globally in model-theoretical terms that are similar to the ones appearing in Lindstrom's theorem.