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Ergodic theorems and metastability: A continuous logic viewpoint

We revisit the Polynomial Mean-Ergodic Theorem (Poly-MET), which asserts the mean convergence of averages of an abelian unitary polynomial action on a Hilbert space. A special case of Walsh's theorem, Poly-MET generalizes the classical ergodic theorem of von Neumann. In this talk we focus on actions of the group $(\mathbb{Z}, +)$. We introduce the class of PET structures (after Bergelson's technique of "Polynomial Ergodic-Theoretic (PET)" inductive descent). A PET structure is a Hilbert space $\mathcal{H}$ endowed with a collection $\mathcal{P}$ of polynomial sequences in $\mathsf{U}(\mathcal{H})$, plus a few other ingredients (notably, a Følner sequence $\{\mathcal{F}_n\}$ and the corresponding averaging operations AV_n). The class of all PET structures with a given upper bound d on the degree of the sequences is axiomatizable in a suitable Henson language, so it is an elementary class of Henson structures. By working in saturated PET structures, it becomes possible to formalize (à la Loeb) classical measure-theoretical arguments, leading to a succinct proof of Poly-MET. Exploiting the compactness of Henson's logic as captured by a "uniform metastability principle" anticipated by Avigad and Iovino, the convergence result admits an immediate refinement to a statement about uniformly metastable convergence. Our approach owes much to Tao's outline of a nonstandard analysis proof of Walsh's theorem. Ongoing work formalizes multiple ergodic averages of polynomial actions of nilpotent groups in the framework of Henson structures.