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A way of obtaining free group algebras inside division rings

In the mid eighties, L. Makar-Limanov conjectured:

- Let D be a division ring with center Z. If D is finitely generated (as a division ring) over Z and $[D : Z] = \infty$, then D contains a noncommutative free Z-algebra.

In many examples for which the conjecture holds, the division ring D contains a (noncommutative) free group algebra over Z, not only a free Z-algebra. Note that if X is the set of free generators of a free algebra, X need not be a set of free generators of a free group algebra.

Let D be a division ring and $(G,<)$ be a (bi)ordered group. Let ∞ be a symbol and extend the operations and ordering on G in the natural way. A *valuation* with values on $(G,<)$ is a map $v\colon D \to G \cup \{\infty\}$ that satisfies, for all $x, y \in D$,

(a) $v(x) = \infty$ if, and only if, $x = 0$.

(b) $v(x + y) \geq \min\{v(x), v(y)\}$.

(c) $v(xy) = v(x)v(y)$.

For each $g \in G$, the sets $D_{\geq g} = \{x \in D\colon v(x) \geq g\}$ and $D_{>g} = \{x \in D\colon v(x) > g\}$ induce a filtration on D whose associated graded ring is $\operatorname{grad}_v(D) = \bigoplus_{g \in G} \frac{D_{\geq g}}{D_{>g}}$

We show that D contains a free group Z-algebra provided that there exist $x_1, x_2 \in D$ with

(1) $v(x_i) > 1$, $i = 1, 2$.

(2) The elements $x_1 + D_{>v(x_1)}, x_2 + D_{>v(x_2)} \in \operatorname{grad}_v(D)$ generate a free $\operatorname{grad}_v(Z)$-subalgebra of $\operatorname{grad}_v(D)$.

First we prove our result for the case of G a subgroup of the real numbers. Using theory of ordered groups, we prove that the general case can be reduced to the foregoing case.