MIKHAIL KOTCHETOV, Memorial University of Newfoundland
Applications of affine group schemes to the study of gradings by abelian groups

Recall that an affine group scheme over a field $\mathbb{F}$ is a representable functor from the category of associative commutative unital $\mathbb{F}$-algebras to the category of groups. In view of Yoneda's Lemma, the representing object of such a functor carries the structure of a Hopf algebra and, conversely, every commutative Hopf algebra yields an affine group scheme. Any (naïve) algebraic group can be regarded as an affine group scheme, but not conversely.

It is well known that if A is an $\mathbb{F}$-algebra (not necessarily associative) and G is a group then a G-grading $A = \bigoplus_{g\in G} A_g$ is equivalent to an $\mathbb{F}G$-comodule algebra structure on A. If G is abelian then the latter is equivalent to a homomorphism from the affine group scheme G^D, represented by $\mathbb{F}G$, to the automorphism group scheme $\mathbf{Aut}_{\mathbb{F}}(A)$ (which is representable if $\dim A < \infty$). If $\mathbb{F}$ is algebraically closed of characteristic 0 then it is sufficient to consider the algebraic group $\mathrm{Aut}_{\mathbb{F}}(A)$ (the $\mathbb{F}$-points of the scheme $\mathbf{Aut}_{\mathbb{F}}(A)$), but not in general.

In this talk, we will look at some examples where the above approach allows one to classify all G-gradings on A up to isomorphism.