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On projective modules over finite quantum groups

Let $\mathcal{D}$ be the Drinfeld double of the bosonization $\mathfrak{B}(V)\#\Bbbk G$ of a finite-dimensional Nichols algebra $\mathfrak{B}(V)$ over a finite group G. It is known that the simple $\mathcal{D}$-modules are parametrized by the simple modules over $\mathcal{D}(G)$, the Drinfeld double of G. This parametrization can be obtained by considering the head $\mathsf{L}(\lambda)$ of the Verma module $\mathsf{M}(\lambda)$ for every simple $\mathcal{D}(G)$-module λ. We will show that the projective $\mathcal{D}$-modules are filtered by Verma modules and the BGG Reciprocity $[\mathsf{P}(\mu) : \mathsf{M}(\lambda)] = [\mathsf{M}(\lambda) : \mathsf{L}(\mu)]$ holds for the projective cover $\mathsf{P}(\mu)$ of $\mathsf{L}(\mu)$. Analogous results are well-known for highest weight categories. However, the categoy of $\mathcal{D}$-modules is not highest weight.

We shall use graded characters to obtain the BGG Reciprocity as consequence of a graded version of it. As a by-product we will show that a Verma module is simple if and only if it is projective. Also, we will describe the tensor product between projective modules.