LORENZO FOSCOLO, Stony Brook University
Non-compact G2 manifolds from asymptotically conical Calabi-Yau 3-folds

G2 manifolds are the Riemannian 7-manifolds with G2 holonomy. Every G2 manifold is necessarily Ricci-flat. Only four examples of complete non-compact G2 manifolds are currently known. In joint work with Mark Haskins and Johannes Nordström we construct infinitely many families of new complete non-compact G2-holonomy manifolds. The underlying smooth 7-manifolds are all circle bundles over asymptotically conical (AC) Calabi-Yau manifolds of complex dimension 3. The metrics are circle-invariant and their geometry at infinity is that of a circle bundle over a Calabi-Yau cone with fibres of fixed finite length. The G2 manifolds we construct are therefore 7-dimensional analogues of 4-dimensional ALF hyperkähler metrics.

The dimensional reduction of the equations for G2 holonomy in the presence of a Killing field was considered by Apostolov-Salamon and by several groups of physicists. We reinterpret the dimensionally-reduced equations in terms of a pair consisting of an SU(3) structure on the 6-dimensional orbit space coupled to an abelian Calabi-Yau monopole on this 6-manifold. We solve this coupled system of non-linear PDEs by considering the adiabatic limit in which the circle fibres of the associated circle-invariant G2-holonomy metrics collapse. The G2-holonomy metrics we construct should be thought of as arising from abelian Hermitian-Yang-Mills connections on AC Calabi-Yau 3-folds, especially AC Calabi-Yau metrics on crepant resolutions of Calabi-Yau cones. All our examples provide instances of families of G2-holonomy metrics that collapse with bounded curvature to Calabi-Yau 3-folds. This collapse with globally bounded curvature is a new feature of G2-holonomy metrics compared to the 4-dimensional ALF hyperkähler setting.