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Non-optimal levels of reducible mod l Galois representations

Let $l \geq 5$ be a prime number and let $\rho : Gal(\overline{\mathbb{Q}}/\mathbb{Q}) \rightarrow GL_2(\overline{\mathbb{F}}_l)$ be a semisimple, odd, continuous, Galois representation. When ρ is irreducible, deep works by many people, culminating in Khare and Wintenberger's proof of Serre's conjectures, ensure that ρ is modular and that the level of a modular form giving rise to it can be taken to be equal to the Serre conductor of ρ. Such level is 'optimal' in that every other prime-to-l admissible level is a multiple of it. Moreover, Diamond and Taylor gave a complete classification of such admissible multiples, that were called 'non-optimal levels'.

In this talk, we address the case where ρ is semisimple, odd, continuous and $reducible$. The modularity in this case can be established by elementary methods, that boil down to study congruences between Eisenstein series and cuspidal Hecke eigenforms. An important difference is that the Serre conductor is not always an admissible level. We will present partial results on criteria for the existence of the optimal level and on level-rasing theorems toward the classification of the corresponding non-optimal levels. Such results are valid in weight $k \geq 4$ and can be used to estimate the degree of the field of coefficients of newforms.

This is joint work with Nicolas Billerey.