Abelian surfaces with quaternionic multiplication, and rational points on Atkin-Lehner quotients of Shimura curves

Let $D$ be the product of an even number of primes and $N$ an integer prime to $D$. We are interested in two problems: proving for large families of pairs $(D, N)$ the triviality of rational points of Atkin-Lehner’s quotients of Shimura curves of discriminant $D$ and level $N$, and proving for large families of pairs $(D, N)$, that there is no geometrically simple abelian surface $A/\mathbb{Q}$ with multiplication, over a quadratic imaginary field, by a maximal order $O_D$ in a quaternion algebra of discriminant $D$ and endowed with a rational isogeny of degree $N^2$ with the kernel $O_D$-cyclic and isomorphic to $(\mathbb{Z}/N\mathbb{Z})^2$. We shall speak about two results related with these problems: First: let $p, q$ be prime numbers. We consider the quotient of the Shimura curve $X^p_{0}(1)$, of discriminant $pq$ and level 1, by the Atkin-Lehner involution $w_q$. We show that the quotient of $X^p_{0}$ by $w_q$ has no rational point for $q > 245$ and $p$ large enough compared to $q$, in the “cas non ramifié de Ogg” $p \equiv 1 \pmod{4}$ and $q \equiv 3 \pmod{4}$ and $\left(\frac{p}{q}\right) = -1$. Second: for a fixed quaternion algebra $B_D$ of discriminant $D$ and a fixed quadratic imaginary field $K$, we find an effective bound for prime $l$ such that there exists a $\Gamma_0(l)$ level structure over $GL_2$-type geometrically simple abelian surfaces $A/\mathbb{Q}$ having multiplication by a maximal order of $B_D$ over $K$. 

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