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On the cohomology of mapping class groups and configuration spaces of non-orientable surfaces

For an orientable surface S_g , characteristic classes of S_g -bundles are given by the cohomology of the classifying space of $\text{Diff}^+(S_g)$, the group of orientation preserving diffeomorphisms of S_g . Moreover, for $g \geq 2$ it is well known that $H^*(B\text{Diff}^+(S_g)) = H^*(\Gamma_g)$, where Γ_g is the mapping class group of S_g . In this work we look at the analogues for non-orientable surfaces. For instance, in the case of the Klein bottle K we show the cohomology of $B\text{Diff}(K)$ is given by the cohomology of $\mathbb{Z}/2 \times \mathbb{Z}/2$ with twisted coefficients in $H^*(\mathbb{C}P^\infty)$ and determine its homotopy type. We also provide concrete Eilenberg-MacLane spaces $K(\pi, 1)$ for the mapping class groups with marked points $\Gamma^q(\mathbb{R}P^2)$ and $\Gamma^q(K)$ and use them to express the cohomology of these groups in terms of the cohomology of configuration spaces. Further relations to mapping spaces are given. This is joint work with C. Hidber and M. Maldonado.