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Isometric Immersions of Pseudo-Spherical Surfaces via Differential Equations

Pseudo-spherical surfaces are surfaces of constant negative Gaussian curvature. A way of realizing such a surface in 3d space as a surface of revolution is obtained by rotating the graph of a curve called tractrix around the z-axis (infinite funnel). There is a remarkable connection between the solutions of the sine-Gordon equation $u_{xt} = \sin u$ and pseudo-spherical surfaces, in the sense that every generic solution of this equation can be shown to give rise to a pseudo-spherical surface. Furthermore, the sine-Gordon equation has the property that the way in which the pseudo-spherical surfaces corresponding to its solutions are realized geometrically in 3d space is given in closed form through some remarkable explicit formulas. The sine-Gordon equation is but one member of a very large class of differential equations whose solutions likewise define pseudo-spherical surfaces. These were defined and classified by Chern, Tenenblat and others, and include almost all the known examples of "integrable" partial differential equations. This raises the question of whether the other equations enjoy the same remarkable property as the sine-Gordon equation when it comes to the realization of the corresponding surfaces in 3d space.

We will see that the answer is no, and will provide a full classification of second-order hyperbolic and kth-order evolution equations. The classification results will show, among other things, that the sine-Gordon equation is quite unique in this regard amongst all integrable equations.