Let $K$ be a simplicial complex. Suppose the vertices of $K$ are painted with $I = \{1, \ldots, m\}$ colours. A homological Sperner-type theorem concludes the existence of a rainbow simplex of $K$ under the hypothesis that certain homology groups of certain subcomplexes of $K$ are zero. The problem of the existence of a rainbow simplex of $K$ is equivalent to the problem of the existence of a system of distinct representatives in a family of sets.