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On the real linear algebra of binary vectors

We are interested in understanding the real linear relations among all 0-1 vectors in $\mathbb{R}^n$, i.e., the linear matroid over $\mathbb{R}$ on the set of $2^n - 1$ nonzero n-vectors whose coordinates are 0 or 1. This fundamental combinatorial object is at the root of questions that have arisen in a variety of fields, from economics to circuit theory to quantum physics, over the past 50 years, a period spanning the development of modern enumerative combinatorics. Yet there has been little real progress in understanding some of the most basic questions here.

In particular, in many applications it is of interest to know the number of regions in $\mathbb{R}^n$ determined by the $2^n - 1$ linear hyperplanes having 0-1 normals. This number can be obtained from the characteristic polynomial of the geometric lattice of all subspaces in $\mathbb{R}^n$ spanned by these 0-1 vectors. These characteristic polynomials are known only through $n = 7$, while just the number of regions is known for $n = 8$.

We discuss the various contexts in which this question has arisen and describe various general approaches to compute the characteristic polynomial, the most promising at the moment being counting the faces of the corresponding broken circuit complex. This is helped by the fact that while this matroid is not truly "binary" (i.e., representable over the field of 2 elements), it is close (it has a binary matroid as a weak image). Along these lines, we give some very partial results toward a general solution.