We prove that, just like polynomials of degree at most $t - 1$, paths along two directions (vertical and horizontal) with at most $t - 1$ turns can simultaneously split any $t$ measures. This is done by analysing the topology of a specific space of possible partitions. Then we look at other problems, like a generalisation of the necklace theorem, that may be approached with similar ideas.