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Lie identities of symmetric Poisson algebras

We recall results on existence of identical relations in group rings and universal (restricted) enveloping algebras of (restricted) Lie (super)algebras in case of different characteristics.

Now we consider related results on identities in Poisson algebras. Let L be a Lie algebra over a field of characteristic $p > 0$. Consider its symmetric algebra $S(L) = \oplus_{n=0}^{\infty} U_n/U_{n-1}$, which is isomorphic to a polynomial ring. It also has a structure of a Poisson algebra, where the Lie product is traditionally denoted by $\{\ ,\ \}$. This bracket naturally induces the structure of a Poisson algebra on the truncated polynomial ring $\mathbf{s}(L) = S(L)/(x^p \mid x \in L)$, which we call a *truncated symmetric Poisson algebra*. We study Lie identical relations of $\mathbf{s}(L)$. Namely, we determine necessary and sufficient conditions for L under which $\mathbf{s}(L)$ is Lie nilpotent, strongly Lie nilpotent, solvable and strongly solvable, where we assume that $p > 2$ to specify the solvability. We compute the strong Lie nilpotency class of $\mathbf{s}(L)$. Also, we prove that the Lie nilpotency class coincides with the strong Lie nilpotency class in case $p > 3$.

Shestakov proved that the symmetric algebra $S(L)$ of an arbitrary Lie algebra L satisfies the identity $\{x, \{y, z\}\} \equiv 0$ if, and only if, L is abelian. We extend this result for the (strong) Lie nilpotency and the (strong) solvability of $S(L)$. We show that the solvability of $\mathbf{s}(L)$ and $S(L)$ in case $\mathrm{char} K = 2$ is different to other characteristics, namely, we construct examples of such algebras which are solvable but not strongly solvable.