On the arithmetic difference of middle Cantor sets

We discuss about the arithmetic difference of affine Cantor sets defined by the simplest possible combinatorics. We determine all triples \((\alpha, \beta, \lambda)\) such that \(C_\alpha - \lambda C_\beta\) forms a closed interval, where \(C_\alpha\) and \(C_\beta\) are middle Cantor sets. In the case \(\lambda = -1\), we extend a result of Mendes and Oliveira. Among the other results, prototype examples of middle Cantor sets \(C_\alpha\) and \(C_\beta\) are introduced such that the sets \(C_\alpha \cdot C_\beta\) and \(C_\alpha / C_\beta\) contain an interval, while the product of their thickness is smaller than one. In sequel, a new family of triples \((C_\alpha, C_\beta, \lambda)\) is indicated for which \(C_\alpha - \lambda C_\beta\) contains an interval or has zero Lebesgue measure. A special triple \((C_\alpha, C_\beta, \lambda)\) is selected and the iterated function system corresponding to the attractor \(C_\alpha - \lambda C_\beta\) has been characterized. Some specifications of the attractor has been presented that keeps our example as an exception among others.