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The three edge theorem

For a k-uniform hypergraph F let $\mathrm{ex}(n, F)$ be the maximum number of edges of a k-uniform n-vertex hypergraph H which contains no copy of F. Determining or estimating $\mathrm{ex}(n, F)$ is a classical and central problem in extremal combinatorics. While for $k = 2$ this problem is well understood, due to the work of Turán and of Erdős and Stone, only very little is known for k-uniform hypergraphs for $k > 2$. We focus on the case when F is a k-uniform hypergraph with three edges on $k+1$ vertices. Already this very innocent (and maybe somewhat particular looking) problem is still wide open even for $k = 3$.

We consider a variant of the problem where the large hypergraph H enjoys additional hereditary density conditions. Questions of this type were suggested by Erdős and Sós about 30 years ago. We show that every k-uniform hypergraph H with density $> 2^{1-k}$ with respect to every large collection of k-cliques induced by sets of $(k-2)$-tuples contains a copy of F. The required density 2^{1-k} is best possible as higher order tournament constructions show.

Our result can be viewed as a common generalisation of the first extremal result in graph theory due to Mantel (when $k = 2$ and the hereditary density condition reduces to a normal density condition) and a recent result of Glebov, Král', and Volec (when $k = 3$ and large subsets of vertices of H induce a subhypergraph of density $> 1/4$). Our proof for arbitrary $k \geq 2$ utilises the regularity method for hypergraphs.