Recently, Chapuy (arXiv 1603.07714) posed an intriguing conjecture regarding Voronoi cells in large random maps: for any non-negative $g$ and any $k > 1$, the masses of the $k$ nearest-neighbour Voronoi cells induced by $k$ uniform points in the genus $g$ Brownian map have the same law as a uniform $k$-division of the unit interval.

Motivated by this conjecture, we show that masses of Voronoi cells are distributed as uniform partitions of $[0, 1]$ for a range of continuum limits of random graphs and maps, including:

1. The Brownian continuum random tree (CRT)
2. The Brownian unicellular map on any fixed (orientable or non-orientable) surface
3. Continuum random graphs with a fixed surplus

Joint work in progress with Omer Angel, Guillaume Chapuy, Éric Fusy, and Christina Goldschmidt.