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*The codegree threshold of  $K_4^-$*

The codegree threshold  $\text{ex}_2(n, F)$  of a non-empty 3-uniform hypergraph  $F$  is the minimum  $d$  such that every 3-uniform hypergraph on  $n$  vertices in which every pair of vertices is contained in at least  $d+1$  edges contains a copy of  $F$  as a subgraph. In this talk, we focus on the codegree threshold of  $F = K_4^-$ , the 3-uniform hypergraph on 4 vertices with 3 edges.

Using flag algebra techniques, we prove that

$$\text{ex}_2(n, K_4^-) = \frac{n}{4} + O(1).$$

This settles in the affirmative a conjecture of Nagle from 1999. In addition, we show that for every near-extremal configuration  $G$ , there is a quasirandom tournament  $T$  on the same vertex-set such that  $G$  is close in the so-called edit distance to the 3-uniform hypergraph  $C(T)$ , whose edges are the cyclically oriented triangles from  $T$ . We further determine the exact value of  $\text{ex}_2(n, K_4^-)$  for infinitely many values of  $n$  by exploiting its very close relation to the existence of skew Hadamard matrices. In fact, we show that determining the exact value of  $\text{ex}_2(n, K_4^-)$  for  $n = 4k + 3$  is equivalent to Seberry's conjecture, which states that there exists a skew Hadamard matrix for every  $n = 4k$ .

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