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Exponential Patterns in Arithmetic Ramsey Theory

If the positive integers are partitioned into finitely many parts $\mathbb{N} = A_1 \cup \cdots \cup A_k$ what can be said about the arithmetic structure of the parts? While it is perhaps unclear that anything can be said about such a general partition, the rich field of arithmetic Ramsey theory is devoted precisely to this question. For example, in 1916 Schur proved that if the positive integers are partitioned into finitely many parts then one of the parts must contain integers x, y and their sum $x + y$. In subsequent years this field has enjoyed many advances and connections to other fields (such as ergodic theory, Fourier analysis, topology) but many difficult questions still remain.

In this talk I'll discuss a host of new results about exponential patterns that appear in every finite partition of the integers. These results are perhaps surprising in that these patterns are extremely "sparse" - in sharp contrast with known results. For example, we show that for every finite partition of the integers there exists $a, b > 1$ so that the triple $\{a, b, a^b\}$ is entirely contained in a part of the partition.