DAVID CONLON, University of Oxford

Unavoidable patterns in words

A word w is said to contain the pattern P if there is a way to substitute a nonempty word for each letter in P so that the resulting word is a subword of w. Bean, Ehrenfeucht and McNulty and, independently, Zimin characterised the patterns P which are unavoidable, in the sense that any sufficiently long word over a fixed alphabet contains P. Zimin's characterisation says that a pattern is unavoidable if and only if it is contained in a Zimin word, where the Zimin words are defined by $Z_1 = x_1$ and $Z_n = Z_{n-1}x_nZ_{n-1}$. We study the quantitative aspects of this theorem, obtaining essentially tight tower-type bounds for the function $f(n,q)$, the least integer such that any word of length $f(n,q)$ over an alphabet of size q contains Z_n. When $n = 3$, the first non-trivial case, we determine $f(n,q)$ up to a constant factor, showing that $f(3,q) = \Theta(2^q q!)$.

Joint work with Jacob Fox and Benny Sudakov.