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On conservation laws of Navier-Stokes Galerkin discretizations

We study conservation properties of Galerkin methods for the incompressible Navier-Stokes equations, without the divergence constraint strongly enforced. In typical discretizations such as the mixed finite element method, the conservation of mass is enforced only weakly, and this leads to discrete solutions which may not conserve energy, momentum, angular momentum, helicity, or vorticity, even though the physics of the Navier-Stokes equations dictate that they should. We aim in this work to construct discrete formulations that conserve as many physical laws as possible without utilizing a strong enforcement of the divergence constraint, and doing so leads us to a new formulation that conserves each of energy, momentum, angular momentum, enstrophy in 2D, helicity and vorticity (for reference, the usual convective formulation does not conserve most of these quantities). Several numerical experiments are performed, which verify the theory and test the new formulation.