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Bijjective Proof of Kasteleyn's Toroidal Perfect Matching Cancellation Theorem

Kasteleyn (1961) showed how to express the dimer partition function for the square lattice on the torus using 4 Pfaffians and stated without proof that the partition function for a graph of genus g requires 2^{2g} Pfaffians (a fact rigorously established almost four decades later).

Via algebraic considerations, Kasteleyn also showed that for the square lattice, under some mild conditions on the edge weights, one of the 4 Pfaffians vanishes (equals 0). It is not evident if and how a similar situation occurs for square lattices of genus $g > 1$. We give a combinatorial proof of Kasteleyn's observation for $g = 1$. We also discuss potential extensions of our result for the higher order genus case and briefly elaborate on possible implications concerning the widely believed assumption in theoretical physics that dimer models are discrete analogues of free fermions.

Based on joint work with A. Jiménez (U. Valparaíso, Chile) and M. Loeb (Charles U., Czech Republic)