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The existence of zero-sum subgraphs in $\{-1, 1\}$-weightings of $E(K_n)$

For a given graph $H$, and a $\{-1, 1\}$-weighting function $f$ of the edges of the complete graph $K_n$, we study the existence of a zero-sum copy of $H$; that is, a copy of $H$ in $K_n$ with $\sum_{e \in E(H)} f(e) = 0$. For example, for all $n \geq 5$, we can guarantee the existence of a zero-sum copy of $K_4$ for every $f : E(K_n) \rightarrow \{-1, 1\}$ if we are provided with enough edges of both types; however, this phenomenon is not true if $H$ is a complete graph on $m \geq 5$ vertices. So, given a graph $H$, our aim is to determine a function $h(H, n)$, if it exists, such that, for sufficiently large $n$, every $f : E(K_n) \rightarrow \{-1, 1\}$ with $\min\{e(-1), e(1)\} \geq h(H, n)$ contains a zero-sum copy of $H$, where $e(-1)$ and $e(1)$ denote the number of edges assigned $-1$ and $1$, respectively. In the cases where $H$ is a path, a star or a tree, the exact function $h(H, n)$ is determined and the extremal $\{-1, 1\}$-weighting functions are characterized.

Of course, such type of problems can be regarded as edge coloring problems: while Ramsey Theory studies the existence of monochromatic subgraphs in edge colorings on the complete graph, and anti-Ramsey Theory studies the existence of rainbow subgraphs, here we study the existence of balanced subgraphs, where balanced means to have the same number of edges of each color.

This is a joint work with Yair Caro and Adriana Hansberg.