A packing $k$-coloring of a graph $G$ is a partition of $V(G)$ into sets $V_1, \ldots, V_k$ such that for each $1 \leq i \leq k$, the distance between any two distinct $x, y \in V_i$ is at least $i + 1$. The packing chromatic number, $\chi_p(G)$, of a graph $G$ is the minimum $k$ such that $G$ has a packing $k$-coloring. Sloper showed that there are 4-regular graphs with arbitrarily large packing chromatic number. The question whether the packing chromatic number of subcubic graphs is bounded appears in several papers. We answer this question in the negative. Moreover, we show that for every fixed $k$ and $g \geq 2k + 2$, almost every $n$-vertex cubic graph of girth at least $g$ has the packing chromatic number greater than $k$. This is joint work with J. Balogh and X. Liu.