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Bounded Solutions to a Singular Parabolic System

Recently, authors have studied singular elliptic systems, such as

$$\begin{cases} -\Delta u = \frac{1}{u^p} + \frac{1}{v^q} \\ -\Delta v = \frac{1}{u^r} + \frac{1}{v^s}, & x \in \Omega, \\ u(x,t) = v(x,t) = 0, & x \in \partial\Omega, \end{cases}$$

in a smooth, bounded domain $\Omega \subset \mathbb{R}^N$. Such problems arise in the study of enzyme kinetics, chemical reactions that are catalyzed by enzymes, as well as some applications to physics used to describe the gravitational potential of self-gravitating, spherically symmetric, polytropic fluid.

This presentation concerns the recently submitted joint work with S. Chen and R. Xu on the following singular parabolic system

$$\begin{cases} u_t = \Delta u + \frac{f(x)}{v^p} \\ v_t = \Delta v + \frac{g(x)}{u^q}, & t > 0, x \in \Omega, \\ u(x,t) = v(x,t) = 0, & x \in \partial\Omega, \end{cases}$$

a natural extension of the elliptic problem found above. Under suitable conditions for real valued constants $p, q > 0$ and the functions $f(x)$, $g(x)$, existence of weak and classical solutions are obtained using a powerful functional method. Such equations are interesting due to their singular nature on the boundary $\partial\Omega$. Applications of classical methods are unsuccessful, so alternative methods must be utilized. Through obtaining bounds for functions combining the solutions u, v, and ϕ, the first normalized eigenfunction of $-\Delta$, we obtain uniform L^p bounds for $u_\varepsilon, v_\varepsilon$ in a related perturbation problem. Then, we use Sobolev embedding theorem to obtain classical solutions.