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Shape and Topological Derivatives via One Sided Differentiation of Lagrangian Functionals

The generic notions of shape and topological derivatives have proven to be both pertinent and useful from the theoretical and numerical points of view. The shape derivative is a differential while the topological derivative often obtained by the method of matched and compound expansions is only a semidiferential (one sided directional derivative). This arises from the fact that the tangent space to the underlying metric spaces of geometries is only a cone. In my recent work the definition of a topological derivative is extended to perturbations obtained by creating holes around curves, surfaces, and microstructures by using the d-dimensional Minkowski content and sets of positive reach. In that context, the Hadamard semidifferential that retains the advantages of the standard differential calculus including the chain rule and the fact that semiconvex functions are Hadamard semidifferentiable is a natural notion to study the semidifferentiability of objective functions with respect to the sets/geometries that belong to complete non-linear non-convex metric spaces.

An important advantage for state constrained objective functions is that theorems on the one-sided differentiation of minimax of Lagrangians can be used to get the semidifferential. For instance, a standard approach to the minimization of a state constrained objective function in Control/ Shape Optimization problems is to consider the minimax of the associated Lagrangian. By using the new notion of averaged adjoint introduced by Sturm and new conditions by Delfour-Sturm, the minimax problem need not be related to a saddle point: non-convex objective functions and non-linear state equations can be directly considered.