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Sufficient conditions for the boundary controllability of the wave equation with transmission conditions

Let $T > 0$ and $\Omega, \Omega_2 \subset \mathbf{R}^2$ be open, connected and strictly convex domains of smooth boundaries $\partial\Omega, \partial\Omega_2$ and such that $\overline{\Omega_2} \subset \Omega$. Let $\Omega_1 := \Omega \setminus \overline{\Omega_2}$. We define the wave equation with transmission conditions by

$$\begin{cases} (\partial_{tt} - c_i^2 \Delta) u^i(x,t) = 0, & (x,t) \in \Omega_i \times (0,T), \\ u^1(x,t) = u^2(x,t), & (x,t) \in \partial\Omega_2 \times (0,T), \\ c_1^2 \partial_{n_2} u^1(x,t) = -c_2^2 \partial_{n_2} u^2(x,t), & (x,t) \in \partial\Omega_2 \times (0,T), \\ u^1(x,t) = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u^i(x,0) = u_0^i(x), u_t^i(x,0) = u_1^i(x), & (x,t) \in \Omega_i \times (0,T), \end{cases}$$

where $c_i > 0, i = 1,2$, n_2 is the outward unit normal of Ω_2 and $(u_0^1 \mathbf{1}_{\Omega_1} + u_0^2 \mathbf{1}_{\Omega_2}, u_1^1 \mathbf{1}_{\Omega_1} + u_1^2 \mathbf{1}_{\Omega_2}) \in H_0^1(\Omega) \times L^2(\Omega)$. We consider the boundary observability of the wave equation with transmission conditions : for $\Gamma \subset \partial\Omega$, does there exist $T > 0$ and $C_T > 0$ such that

$$E(u^1, u^2)(t) \leq C_T \int_0^T \int_\Gamma \left|\partial_n u^1(x,t)\right|^2 \mathrm{dxdt}$$

holds? It is known that the observability fails to hold if $c_2 < c_1$ since there are rays of the optic geometry that cannot leave Ω_2. Thus, consider the case where $c_2 > c_1$, the case $c_2 = c_1$ reducing to the observability of the classical wave equation.

We prove the following. Let $x_0 \in \mathbf{R}^2 \setminus \overline{\Omega}$ and $\Gamma(x_0) := \{x \in \partial\Omega \,|\, (x - x_0).n(x) > 0\}$. If the flow induced by the boundaries $\partial\Omega_2$ and $\partial\Omega \setminus \Gamma(x_0)$ is dispersive, then for any $c_2 > c_1$, there exist $T > 0$ such that the observability holds. Otherwise, there exists $c^* > 1$ such that for every $c^* > c_2/c_1$, there exists $T > 0$ such that the observability holds.