
ALBERTO MERCADO, Universidad Técnica Federico Santa María
Controllability of wave equations in heterogeneous media

We present a controllability problem for semilinear wave equations with discontinuous main coefficient:

$$u_{tt} - \operatorname{div}(a\nabla u) + g(u) = f \quad (1)$$

These equations appear as a model of wave propagation in heterogeneous media. We deal with the case of locally constant main coefficients with a jump in the boundary of a interior subdomain Ω_1 :

$$a(x) = \begin{cases} a_1 & x \in \Omega_1 \\ a_2 & x \in \Omega \setminus \overline{\Omega_1} \end{cases}$$

with $a_j > 0$ for $j = 1, 2$.

Then we consider the equation as a pair of wave equation with constant coefficients a_1, a_2 , coupled with the so-called transmission conditions

$$\begin{cases} u_1 = u_2 & \text{on } \partial\Omega_1 \\ a_1 \frac{\partial u_1}{\partial n_1} + a_2 \frac{\partial u_2}{\partial n_2} = 0 & \text{on } \partial\Omega_1. \end{cases} \quad (2)$$

We will show controllability results when the Ω_1 is convex and some generalizations. The main tool is the use of adequate Carleman estimates. Also, we will present some related problems regarding models for wave propagation in beams and some inverse problems.