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Generalized complex numbers over near-fields

In the early 20th century, Dickson (1905) investigated the redundancy or not of the field axioms. By a clever disturbance of the multiplication of a field, he demonstrated the existence of an algebraic structure fulfilling all the requirements of a field except one of the distributive axioms. These structures are known as Dickson near-fields, but there are many near-fields not of this type. Almost immediately near-fields were shown to be not just an algebraic curiosity. Veblen and Wedderburn (1907) showed that near-fields are exactly the algebraic structures required for coordinization of geometries that lead to non-Desarguesian planes. In a monumental paper, Zassenhaus (1935/6) showed that all finite near-fields are Dickson near-fields except for 7 strays. There are many other applications of near-fields and the more general near-rings became an important and useful area of investigation with its own concerns and problems catering for non-linear algebraic systems.

The construction of the complex numbers over the reals has been generalized in many ways leading to the 2-dimensional elliptical complex numbers (= complex numbers) and the parabolic and hyperbolic complex numbers. These can be extended to higher dimensions and using an arbitrary ring as the base ring.

It is possible to define matrices and polynomials over near-rings. Using these, one can construct generalized complex numbers over a near-field. In this talk, this construction will be formalized. We also report on properties of this algebraic structure and highlight similarities and differences with its motivating example; the usual complex numbers over the real field.